

Chapter 7. Coordinate Geometry

Question-1

Find the centroid of the triangle whose vertices are : (3, -5), (-7, 4), (10, -2).

Solution:

The centroid of the triangle whose vertices are (3, -5), (-7, 4), (10, -2) is

$$\therefore \text{The centroid of the triangle ABC} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$\therefore \text{The centroid of the triangle whose vertices A(3, -5), B(-7, 4), C(10, -2) = } \left(\frac{3 - 7 + 10}{3}, \frac{-5 + 4 - 2}{3} \right) = (2, -1).$$

Question-2

Find the centroid of the triangle whose vertices are : (2, 1), (5, 2), (3, 4).

Solution:

$$\therefore \text{The centroid of the triangle ABC} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$\therefore \text{The centroid of the triangle whose vertices (2, 1), (5, 2), (3, 4) = } \left(\frac{2+5+3}{3}, \frac{1+2+4}{3} \right) = (10/3, 7/3).$$

Question-3

Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Given, the two vertices of the triangle are (-3, 1), (0, -2).

Let the third vertex be (x, y)

Also given the centroid of the triangle = (0, 0)

$$\therefore \text{The centroid of the triangle} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$\therefore \left(\frac{-3 + 0 + x}{3}, \frac{1 - 2 + y}{3} \right) = (0, 0)$$

$$\Rightarrow -3 + x = 0, -1 + y = 0$$

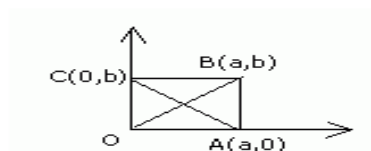
$$\therefore x = 3, y = 1$$



Question-4

Prove that the diagonals of a rectangle bisect each other and are equal.
(Hint: With O as origin, let the vertices of the rectangle be (0, 0), (a, 0), (a, b) and (0, b)).

Solution:



AC and OB are diagonals

In the figure let the intersecting point of OB and AC be P

To show that diagonals bisect each other we have to prove that $OP = PB$ and $PA = PC$

The co-ordinates of P is obtained by

$$\left(\frac{0+a}{2}, \frac{0+b}{2} \right)$$

$\therefore$ P is the point $\left(\frac{a}{2}, \frac{b}{2} \right)$

$$\begin{aligned} OP^2 &= \left(\frac{a}{2} - 0 \right)^2 + \left(\frac{b}{2} - 0 \right)^2 \\ &= \frac{a^2}{4} + \frac{b^2}{4} \quad \therefore OP = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \end{aligned}$$

$$\begin{aligned} PB^2 &= \left(a - \frac{a}{2} \right)^2 + \left(b - \frac{b}{2} \right)^2 \\ &= \frac{a^2}{4} + \frac{b^2}{4} \quad PB = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \end{aligned}$$

$$OP = PB$$

Similarly we can prove that $PC = PA$

Thus diagonals bisect each other in a rectangle.

$$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - a)^2 + (b - 0)^2} \\ &= \sqrt{b^2 + a^2} \end{aligned}$$

$$\begin{aligned} OB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(a - 0)^2 + (0 - b)^2} \\ &= \sqrt{a^2 + b^2} \end{aligned}$$

$$AC = BO$$

$\therefore$ The diagonals of a rectangle bisect each other and equal.

Question-5

Show that the points A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are the vertices of a parallelogram. (Hint: Diagonals of a parallelogram bisect each other).

Solution:

The midpoint of diagonal AC is $\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

The midpoint of diagonal BD is $\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

The diagonals AC and BD bisect each other $\Rightarrow$ ABCD is a parallelogram.

Question-6

In what ratio does the y -axis divide the line segment joining the points P(-4, 5) and Q(3, 7)?

Solution:

Let the required ratio be $k : 1$. Then, the coordinates of the point of division are, $\left(\frac{3k - 4}{k + 1}, \frac{7k + 5}{k + 1}\right)$

But, it is a point on y-axis on which x-coordinate of every point is zero.

Therefore $\frac{3k - 4}{k + 1} = 0$

$$3k - 4 = 0$$

$$k = 4/3$$

Thus, the required ratio is $k = 4/3$ or $4 : 3$.

Question-7

Find the circumcentre of the triangle whose vertices are (-2, -3), (-1, 0), (7, -6).

Solution:

Let the centre of the circle be O(x, y). The points are A(-2, -3), B(-1, 0) and C(7, -6).

$$OA = \sqrt{(-2-x)^2 + (-3-y)^2}$$

$$OB = \sqrt{(-1-x)^2 + (0-y)^2}$$

$$OC = \sqrt{(7-x)^2 + (-6-y)^2}$$

$$OA = OB = OC = \text{radius}$$

$$\sqrt{(-2-x)^2 + (-3-y)^2} = \sqrt{(-1-x)^2 + (0-y)^2}$$

$$(2+x)^2 + (3+y)^2 = (1+x)^2 + y^2$$

$$4 + 4x + x^2 + 9 + 6y + y^2 = 1 + 2x + x^2 + y^2$$

$$4 + 4x + 9 + 6y = 1 + 2x$$

$$2x + 6y = -12$$

$$x + 3y = -6 \dots\dots\dots(i)$$

$$\sqrt{(-1-x)^2 + (0-y)^2} = \sqrt{(7-x)^2 + (-6-y)^2}$$

$$(1+x)^2 + y^2 = (7-x)^2 + (6+y)^2$$

$$1 + 2x + x^2 + y^2 = 49 - 14x + x^2 + 36 + 12y + y^2$$

$$1 + 2x = 49 - 14x + 36 + 12y$$

$$16x - 12y = 84$$

$$4x - 3y = 21 \dots\dots\dots(ii)$$

Solving (i) and (ii)

$$x + 3y = -6$$

$$\underline{4x - 3y = 21}$$

$$5x = 15$$

$$x = 3$$

Substituting $x = 3$ in (i)

$$4(3) - 3y = 21$$

$$12 - 3y = 21$$

$$-3y = 9$$

$$y = -3$$

Therefore the centre of the circle is (3, -3).

Question-8

The three vertices of a parallelogram are (1, 1), (4, 4) and (4, 8). Find the fourth vertex.

Solution:

Let A(1, 1), B(4, 4), C(4, 8) and D(x, y) be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

Therefore coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$$\left(\frac{1+4}{2}, \frac{1+8}{2}\right) = \left(\frac{4+x}{2}, \frac{8+y}{2}\right)$$

$$\left(\frac{5}{2}, \frac{9}{2}\right) = \left(\frac{4+x}{2}, \frac{8+y}{2}\right)$$

$$4+x = 5, 8+y = 9$$

$$x = 1, y = 1$$

Therefore (1, 1) is the fourth vertex.

Question-9

Show that the points (2, 1), (5, 2), (6, 4) and (3, 3) are the angular points of a parallelogram. Is the figure a rectangle?

Solution:

Let A(2, 1), B(5, 2), C(6, 4) and D(3, 3) be the vertices of a parallelogram ABCD. Since, the diagonals of a parallelogram bisect each other.

$$AC^2 = (6 - 2)^2 + (4 - 1)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$$

$$BC^2 = (6 - 5)^2 + (4 - 2)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$$

$$AB^2 = (5 - 2)^2 + (2 - 1)^2 = (3)^2 + (1)^2 = 9 + 1 = 10$$

$$DC^2 = (6 - 3)^2 + (4 - 3)^2 = (3)^2 + (1)^2 = 9 + 1 = 10$$

$$AD^2 = (3 - 2)^2 + (3 - 1)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$$

Since $BC = AD$ and $DC = AB$, ABCD is a parallelogram.

$$AB^2 + BC^2 = 10 + 5 = 15$$

$$AB^2 + BC^2 \neq AC^2$$

$\therefore \triangle ABC$ is not right angled. Therefore parallelogram ABCD is not a rectangle.

Question-10

Find the third vertex of a triangle, if two of its vertices are (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Let the third vertex of the triangle be C(x, y) and the other vertices A(-3, 1) and B(0, -2).

Coordinates of the centroid of the triangle = (0, 0)

$$\therefore \left(\frac{-3 + 0 + x}{3}, \frac{1 - 2 + y}{3} \right) = (0, 0)$$

$$-3 + x = 0$$

$$x = 3$$

$$\text{And } -1 + y = 0$$

$$y = 1$$

$\therefore$ The third vertex of the triangle is (3, 1).

Question-11

If the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4), find its vertices.

Solution:

Let the vertices of the triangle be A(x_1 , y_1), B(x_2 , y_2) and C(x_3 , y_3). Let the midpoints of the sides of a triangle are

D(1, 1), E(2, -3) and F(3, 4).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (1, 1)$$

$$x_1 + x_2 = 2 \dots\dots\dots(i)$$

$$y_1 + y_2 = 2 \dots\dots\dots(ii)$$

$$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (2, -3)$$

$$x_2 + x_3 = 4 \dots\dots\dots(iii)$$

$$y_2 + y_3 = -6 \dots\dots\dots(iv)$$

$$\left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2}\right) = (3, 4)$$

$$x_3 + x_1 = 6 \dots\dots\dots(v)$$

$$y_3 + y_1 = 8 \dots\dots\dots(vi)$$

Add the equations (i), (iii), (v) we get,

$$2(x_1 + x_2 + x_3) = 12$$

$$\Rightarrow x_1 + x_2 + x_3 = 6 \dots\dots\dots(vii)$$

Substitute eqn.(i) in (vii) then $x_3 = 4$.

Substitute eqn.(iii) in (vii) then $x_1 = 2$.

Substitute eqn.(v) in (vii) then $x_2 = 0$.

Add the equations (ii), (iv), (vi) we get,

$$2(y_1 + y_2 + y_3) = 4$$

$$\Rightarrow y_1 + y_2 + y_3 = 2 \dots\dots\dots(viii)$$

Substitute eqn.(ii) in (viii) then $y_3 = 0$.

Substitute eqn.(iv) in (viii) then $y_1 = 8$.

Substitute eqn.(vi) in (viii) then $y_2 = -6$.

$\therefore$ The vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3) = A(2, 8)$, $B(0, -6)$ and $C(4, 0)$

Question-12

Find the ratio in which the line points (6, 4) and (1, - 7) is divided internally by the axis of x.

Solution:

Let the required ratio be $k : 1$. Then, the coordinates of the point of division

are, $\left(\frac{k + 6}{k + 1}, \frac{-7k + 4}{k + 1}\right)$

But, it is a point on y-axis on which x-coordinate of every point is zero.

Therefore $\frac{-7k + 4}{k + 1} = 0$

$$7k - 4 = 0$$

$$k = 4/7$$

Thus, the required ratio is $k = 4/7$ or $4 : 7$.

Question-13

If the points (-2, -1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Solution:

Let the vertices of the parallelogram be A(-2, -1), B(1, 0), C(x, 3) and D(1, y).
Since the diagonals of a parallelogram bisect each other the coordinates of the mid-point of AC = coordinates of the mid-point of BD.

$$\left(\frac{-2+x}{2}, \frac{-1+3}{2}\right) = \left(\frac{1+1}{2}, \frac{0+y}{2}\right)$$

$$\left(\frac{-2+x}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

$$\therefore x = 4, y = 2.$$

Question-14

If the mid-points of the sides of a triangle PQR are A(-1, -3), B(2, 1) and C(4, 5), find the coordinates of P, Q and R.

Solution:

Coordinate of mid point P is $\left(\frac{-1+2}{2}, \frac{-3+1}{2}\right) = \left(\frac{1}{2}, -1\right)$

Coordinate of mid point Q is $\left(\frac{2+4}{2}, \frac{1+5}{2}\right) = (3, 3)$

Coordinate of mid point R is $\left(\frac{-1+4}{2}, \frac{-3+5}{2}\right) = \left(\frac{3}{2}, 1\right)$

$\therefore$ The coordinates of the P $\left(\frac{1}{2}, -1\right)$, Q(3, 3) and R $\left(\frac{3}{2}, 1\right)$.

Question-15

Three consecutive vertices of a parallelogram are (-2, -1), (1, 0) and (4, 3).
Find the fourth vertex.

Solution:

Let A(-2, -1), B(1, 0), C(4, 3) and D(x, y) be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

Therefore coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = \left(\frac{1+x}{2}, \frac{0+y}{2}\right)$$

$$(1, 1) = \left(\frac{1+x}{2}, \frac{y}{2}\right)$$

$$x + 1 = 2, y = 2$$

Therefore (1, 2) is the fourth vertex.

Question-16

Determine the ratio in which $2x + 3y - 30 = 0$ divides the line segment joining A (3, 4) and B (7, 8) and the point at which it divides.

Solution:

Let the P(a, b) be the point which divides the line segment joining A (3, 4) and B (7, 8) in the ratio $k : 1$.

Then coordinates of the point P is $\left(\frac{7k+3}{k+1}, \frac{8k+4}{k+1}\right)$.

This point lies on the line $2x + 3y - 30 = 0$.

$$\therefore 2\left(\frac{7k+3}{k+1}\right) + 3\left(\frac{8k+4}{k+1}\right) - 30 = 0.$$

$$\therefore 2(7k+3) + 3(8k+4) - 30(k+1) = 0.$$

$$\therefore 14k + 6 + 24k + 12 - 30k - 30 = 0.$$

$$\therefore 8k - 12 = 0.$$

$$\therefore k = 3/2$$

$\therefore$ The required ratio is 3 : 2.

The coordinates of the point P is $\left(\frac{21+6}{5}, \frac{24+8}{5}\right) = \left(\frac{27}{5}, \frac{32}{5}\right)$.

Question-17

Prove that the points $(2a, 4a)$, $(2a, 6a)$, $(2a + \sqrt{3}a, 5a)$ are the vertices of an equilateral triangle.

Solution:

Let A(2a, 4a), B(2a, 6a) and C(2a + $\sqrt{3}a$, 5a) be the vertices of an equilateral triangle.

$$AB = \sqrt{(2a-2a)^2 + (6a-4a)^2} = \sqrt{0^2 + (2a)^2} = 2a$$

$$BC = \sqrt{(2a+\sqrt{3}a-2a)^2 + (5a-6a)^2} = \sqrt{3a^2 + a^2} = 2a$$

$$CA = \sqrt{(2a+\sqrt{3}a-2a)^2 + (5a-4a)^2} = \sqrt{(\sqrt{3}a)^2 + a^2} = \sqrt{3a^2 + a^2} = 2a$$

$$AB = BC = CA.$$

$\therefore$ the vertices are of an equilateral triangle.

Question-18

The points A(0, 3), B(-2, a) and C(-1, 4) are the vertices of a ΔABC right-angled at A. Find the value of a.

Solution:

Given, the vertices of a ΔABC are right – angled at A.

$$\therefore AB^2 + AC^2 = BC^2$$

$$AB^2 = (-2 - 0)^2 + (a - 3)^2 = 4 + (a - 3)^2$$

$$BC^2 = (-1 + 2)^2 + (4 - a)^2 = 1 + (4 - a)^2$$

$$AC^2 = (-1 - 0)^2 + (4 - 3)^2 = 1 + 1 = 2$$

$$\text{Since, } AB^2 + AC^2 = BC^2$$

$$4 + (a - 3)^2 + 2 = 1 + (4 - a)^2$$

$$4 + a^2 + 9 - 6a + 2 = 1 + 16 + a^2 - 8a$$

$$2a = 2$$

$$\therefore a = 1.$$

Question-19

The points A(2, 0), B(9, 1), C(11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Solution:

$$AB = \sqrt{(9-2)^2 + (1-0)^2} = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$BC = \sqrt{(11-9)^2 + (6-1)^2} = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$$

$$CD = \sqrt{(4-11)^2 + (4-6)^2} = \sqrt{49 + 4} = \sqrt{53}$$

$$DA = \sqrt{(2-4)^2 + (0-4)^2} = \sqrt{4 + 16} = \sqrt{20}$$

No it is not a rhombus as all sides are not equal.

Question-20

The vertices of a triangle are A(3, 4), B(7, 2) and C(-2, -5). Find the length of the median through the vertex A.

Soln

Let D the mid point of BC. Then the coordinate of D is $\left(\frac{7-2}{2}, \frac{2-5}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$.

$$\begin{aligned} \text{Length of the median is } AD &= \sqrt{\left(\frac{5}{2} - 3\right)^2 + \left(\frac{-3}{2} - 4\right)^2} \\ &= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{-11}{2}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{121}{4}} \\ &= \frac{\sqrt{122}}{2}. \end{aligned}$$

Question-21

Two vertices of a triangle are (3, -5) and (-7, 4). If its centroid is (2, -1), find the third vertex.

Solution:

Let P(3, -5) and Q(-7, 4) be the two vertices of a triangle. Centroid of a triangle is G(2, -1).

The coordinates of the centroid of ΔABC is $\left(\frac{x_1 + 3 - 7}{3}, \frac{y_1 - 5 + 4}{3}\right) = (2, -1)$
 $\left(\frac{x_1 - 4}{3}, \frac{y_1 - 1}{3}\right) = (2, -1)$

$$x_1 = 6 + 4, y_1 = -3 + 1$$

$$x_1 = 10, y_1 = -2$$

$\therefore$ The third vertex is (10, -2).

Question-22

Are the points (-2, 2), (8, -2) and (-4, 3) are the vertices of a right angled triangle.

Solution:

Let the points A(-2, 2), B(8, -2) and C(-4, 3) be the vertices of a triangle ABC.

$$AB^2 = (8 + 2)^2 + (-2 - 2)^2 = 10^2 + 4^2 = 100 + 16 = 116$$

$$BC^2 = (-4 - 8)^2 + (3 + 2)^2 = (-12)^2 + 5^2 = 144 + 25 = 169$$

$$AC^2 = (-4 + 2)^2 + (3 - 2)^2 = (-2)^2 + 1 = 4 + 1 = 5$$

$$AB^2 + AC^2 \neq BC^2$$

The above vertices are not points of a right angled triangle.

Question-23

If (-2, 3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.

Solution:

Let P(-2, 3), Q(4, -3), R(4, 5) be the mid-points of sides AB, BC and CA respectively of a triangle ABC. Let A(x_1 , y_1), B(x_2 , y_2) and C(x_3 , y_3) be the vertices of triangle ABC. Then,
P is the midpoint of AB.

$$\frac{x_1 + x_2}{2} = -2, \frac{y_1 + y_2}{2} = 3$$

$$x_1 + x_2 = -4 \text{ and } y_1 + y_2 = 6 \dots\dots\dots(i)$$

Q is the midpoint of BC.

$$\frac{x_2 + x_3}{2} = 4, \frac{y_2 + y_3}{2} = -3$$

$$x_2 + x_3 = 8 \text{ and } y_2 + y_3 = -6 \dots\dots\dots(ii)$$

R is the midpoint of CA.

$$\frac{x_1 + x_3}{2} = 4, \frac{y_1 + y_3}{2} = 5$$

$$x_1 + x_3 = 8 \text{ and } y_1 + y_3 = 10 \dots\dots\dots(iii)$$

From (i), (ii) and (iii)

$$2(x_1 + x_2 + x_3) = -4 + 8 + 8 = 12$$

$$x_1 + x_2 + x_3 = 6$$



$$2(y_1 + y_2 + y_3) = 6 - 6 + 10 = 10$$

$$y_1 + y_2 + y_3 = 5$$

Therefore the coordinates of the centroid of ΔABC are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

or, $(6/3, 5/3) = (2, 5/3)$.

Question-24

If the points $(-1, 3)$, $(1, -1)$ and $(5, 1)$ are vertices of a triangle, find the length of the median through third vertex.

Solution:

Let the points $A(-1, 3)$, $B(1, -1)$ and $C(5, 1)$ be the vertices of a triangle ABC .

Let D be the mid point of AB . Then the coordinate of D is $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right) = (0, 1)$.

$$\begin{aligned}\text{Length of the median is } CD &= \sqrt{(0-5)^2 + (1-1)^2} \\ &= \sqrt{25 + 0} \\ &= 5.\end{aligned}$$

Question-25

Find the lengths of the sides of the triangle whose vertices are $A(3, 4)$, $B(2, -1)$ and $C(4, -6)$.

Solution:

$$\begin{aligned}AB &= \sqrt{(2-3)^2 + (-1-4)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26} \\ BC &= \sqrt{(4-2)^2 + (-6+1)^2} = \sqrt{(2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29} \\ AC &= \sqrt{(4-3)^2 + (-6-4)^2} = \sqrt{(1)^2 + (-10)^2} = \sqrt{1+100} = \sqrt{101}\end{aligned}$$

Therefore the lengths of the sides of the triangle are $\sqrt{26}$, $\sqrt{29}$ and $\sqrt{101}$.

Question-26

A line is of length 10 and one end is at the point $(-3, 2)$. If the ordinate of the other end be 10, prove that the abscissa will be 3 or -9 .

Solution:

The two points are $(-3, 2)$ and $(x, 10)$. Their length is 10.

$$\sqrt{(x+3)^2 + (10-2)^2} = 10$$

$$(x+3)^2 + 8^2 = 10^2$$

$$x^2 + 6x + 9 + 64 = 100$$

$$x^2 + 6x - 27 = 0$$

$$x^2 + 9x - 3x - 27 = 0$$

$$x(x+9) - 3(x+9) = 0$$

$$(x-3)(x+9) = 0$$

$$x = 3 \text{ or } -9.$$

Therefore the required abscissa will be 3 or -9 .

Question-27

Show that the points $(-2, 6)$, $(5, 3)$, $(-1, -11)$ and $(-8, -8)$ are the vertices of a rectangle.

Solution:

Let $A(-2, 6)$, $B(5, 3)$, $C(-1, -11)$ and $D(-8, -8)$ are the vertices of a rectangle.

$$(i) AB = \sqrt{(5+2)^2 + (3-6)^2} = \sqrt{(7)^2 + (-3)^2} = \sqrt{49+9} = \sqrt{58}$$

$$CD = \sqrt{(-8+1)^2 + (-8+11)^2} = \sqrt{(-7)^2 + 3^2} = \sqrt{49+9} = \sqrt{58}$$

$$BC = \sqrt{(-1-5)^2 + (-11-3)^2} = \sqrt{(-6)^2 + (-14)^2} = \sqrt{36+196} = \sqrt{232}$$

$$DA = \sqrt{(-8+2)^2 + (-8-6)^2} = \sqrt{(-6)^2 + (-14)^2} = \sqrt{36+196} = \sqrt{232}$$

$$AB = CD \text{ and } BC = DA.$$

$$(ii) AC = \sqrt{(-1+2)^2 + (-11-6)^2} = \sqrt{(1)^2 + (-17)^2} = \sqrt{1+289} = \sqrt{290}$$

$$BD = \sqrt{(-8-5)^2 + (-8-3)^2} = \sqrt{(-13)^2 + (-11)^2} = \sqrt{169+121} = \sqrt{290}$$

$$AC = DB$$

Since the opposite sides and the diagonals are equal
 $\therefore$ ABCD is a rectangle.

Question-28

If a point (x, y) is equidistant from $(6, -1)$ and $(2, 3)$, find the relation between x and y .

Solution:

Let the points be $P(x, y)$, $A(6, -1)$ and $B(2, 3)$.

$$AP^2 = (x - 6)^2 + (y + 1)^2$$

$$BP^2 = (x - 2)^2 + (y - 3)^2$$

Given, (x, y) is equidistant from $(6, -1)$ and $(2, 3)$

$$(x - 6)^2 + (y + 1)^2 = (x - 2)^2 + (y - 3)^2$$

$$x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$$

$$-12x + 36 + 2y + 1 = -4x + 4 - 6y + 9$$

$$-8x + 8y = -24$$

$$-x + y = -3$$

$$x - y = 3.$$

Question-29

Coordinates of A and B are $(-3, a)$ and $(1, a + 4)$. The mid-point of AB is $(-1, 1)$. Find the value of a .

Solution:

$$\text{Mid point of AB} = \left(\frac{-3+1}{2}, \frac{a+a+4}{2} \right)$$



$$\left(\frac{-3+1}{2}, \frac{a+a+4}{2}\right) = (-1, 1)$$

$$\left(\frac{-2}{2}, \frac{2a+4}{2}\right) = (-1, 1)$$

$$a + 2 = 1$$

$$a = -1.$$

Question-30

Find a point on the line through A(5, -4) and B(-3, 2), that is, twice as far from A as from B.

Solution:

Let the required point be P(x, y).

Then, AP = 2PB

$$AP/PB = 2/1$$

or AP : PB = 2 : 1

$$\text{Therefore } x = \left[\frac{2 \times (-3) + 1 \times 5}{2+1}\right] = \frac{-1}{3} \text{ and } y = \left[\frac{2 \times 2 + 1 \times (-4)}{2+1}\right] = 0$$

So, the required point is (-1/3, 0).

Question-31

Determine the ratio in which $y - x + 2 = 0$ divides the line joining (3, -1) and (8, 9).

Solution:

Let the required ratio be k : 1.

Then, the point of division is $\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$.

This point must lie on $y - x + 2 = 0$.

$$\text{Therefore } \frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0 \text{ or } k = \frac{2}{3}.$$

So, the required ratio is $\frac{2}{3}:1$

i.e. 2 : 3.

Question-32

If the points (2, 1) and (1, -2) are equidistant from the point (x, y), show that $x + 3y = 0$.

Solution:

Let the points A(2, 1) and B(1, -2) be at equidistant from the point P(X, Y).

$$AP = \sqrt{(x-2)^2 + (y-1)^2}$$

$$AB = \sqrt{(x-1)^2 + (y+2)^2}$$

$$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$$

$$(x-2)^2 + (y-1)^2 = (x-1)^2 + (y+2)^2$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 + 4y + 4$$

$$-4x + 4 - 2y + 1 = -2x + 1 + 4y + 4$$

$$-2x - 6y = 0$$

$$x + 3y = 0.$$

Question-33

Find the ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and $(6, 3)$ and hence find the value of y .

Solution:

Let the required ratio be $k : 1$.

$$\text{Then, } 2 = \frac{6k - 4 \times 1}{k + 1} \Rightarrow k = \frac{3}{2}.$$

Therefore the required ratio is $\frac{3}{2} : 1$ i.e. $3 : 2$.

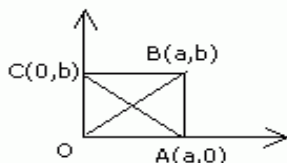
$$\text{Also, } y = \frac{3 \times 3 + 2 \times 3}{3 + 2} = 3.$$

Question-34

Prove that the diagonals of a rectangle bisect each other and are equal.

(Hint: With O as origin, let the vertices of the rectangle be $(0, 0)$, $(a, 0)$, (a, b) and $(0, b)$).

Solution:



ABCO is a rectangle with vertices $A(a, 0)$, $B(a, b)$, $C(0, b)$ and $O(0, 0)$.

The midpoint of AC is $(\frac{a+0}{2}, \frac{0+b}{2}) = (a/2, b/2)$

The midpoint of OB is $(\frac{a+0}{2}, \frac{b+0}{2}) = (a/2, b/2)$

Hence the diagonals bisect each other.

The length of the diagonal AC = $\sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$ units

The length of the diagonal OB = $\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$ units

Hence the diagonals are equal.

Question-35

Show that the points A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are the vertices of a parallelogram. (Hint: Diagonals of a parallelogram bisect each other).

Solution:

The midpoint of diagonal AC is $(\frac{1+2}{2}, \frac{0+7}{2}) = (3/2, 7/2)$

The midpoint of diagonal BD is $(\frac{5-2}{2}, \frac{3+4}{2}) = (3/2, 7/2)$

The diagonals AC and BD bisect each other $\Rightarrow$ ABCD is a parallelogram.

Question-36

If the distances of A(x, y) from P(a + b, b - a) and Q(a - b, a + b) are equal, prove that bx = ay.

Solution:

$$AP^2 = (a + b - x)^2 + (b - a - y)^2$$

$$AQ^2 = (a - b - x)^2 + (a + b - y)^2$$

$$AP = AQ \text{ (Given)}$$

$$\therefore AP^2 = AQ^2$$

$$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$$

$$a^2 + b^2 + x^2 + 2ab - 2ax - 2bx + b^2 + a^2 + y^2 - 2ba - 2by + 2ay = a^2 + b^2 + x^2 - 2ab + 2bx - 2ax + a^2 + b^2 + y^2 + 2ab - 2ay - 2by - 2bx + 2ay = 2bx - 2ay$$

$$4ay = 4bx$$

$$ay = bx$$

Hence proved.

Question-37

Prove that the points A (0, 1), B(1, 4), C(4, 3) and D(3, 0) are the vertices of a square.

Solution:

Let A (0, 1), B(1, 4), C(4, 3) and D(3, 0) be the four points.

$$AB = \sqrt{(1-0)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10} \text{ units}$$

$$BC = \sqrt{(4-1)^2 + (3-4)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$CD = \sqrt{(3-4)^2 + (0-3)^2} = \sqrt{1+9} = \sqrt{10} \text{ units}$$

$$DA = \sqrt{(0-3)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

Therefore the points A (0, 1), B(1, 4), C(4, 3) and D(3, 0) are the vertices of a square.



Question-38

Show that the points (1, 1), (-2, 7) and (3, -3) are collinear.

Solution:

Let A(1, 1), B(-2, 7) and C(3, -3) be the three points.

$$AB = \sqrt{(-2-1)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

$$BC = \sqrt{(3+2)^2 + (-3-7)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5} \text{ units}$$

$$AC = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$AB + AC = BC$. Hence A, B, C are collinear.

Question-39

Find the ratio in which the points (2, 5) divides the line-segment joining the points (-1, 2) and (4, 7).

Solution:

Let A(-1, 2) and B(4, 7) be points. Let the point at which it is divided be C(2, 5).

$$AC^2 = (2+1)^2 + (5-2)^2 = (3)^2 + (3)^2 = 9+9 = 18$$

$$BC^2 = (2-4)^2 + (5-7)^2 = (-2)^2 + (-2)^2 = 4+4 = 8$$

$$AC/BC = 18/8 = 9/4$$

The ratio in which the line segment is divided is 9 : 4.